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An isogeometric error estimate for transport equation is obtained in $2D$

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In this paper, an isogeometric error estimate for transport equation is obtained in $2D$ to prove the convergence of isogeometric method. The result that we have obtained, generalizes Ern result, got in finite elements method (cf. [1, 2]). For the time discretization, the two stage Heun scheme is used to prove this result. For a polynomial of degree $k \geq 1$, the order of convergence in space is 2 and in time is $k + \frac{1}{2}$.
Keywords : error estimate; isogeometric method; the two stage Heun scheme; transport equation

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Analyse numérique du mouvement vibratoire d'une poutre d'Euler-Bernoulli

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Cette étude porte sur l'analyse numérique du mouvement vibratoire d'une poutre d'Euler-Bernoulli contrôlé à ses bords. Le mouvement de cette poutre est régi par les équations aux dérivées partielles (EDPs) suivantes :

$$\begin{cases} w_{tt}(x,t) + w_{xxxx}(x,t) = 0, & 0 < x < 1,\ t \geq 0 \\ w(0,t) = w_x(0,t) = w(1,t) = 0, & t \geq 0 \\ w_{xx}(1,t) = u(t), & t \geq 0 \\ w(x,0) = w_0(x),\ w_t(x,0) = w_1(x), & 0 \leq x \leq 1 \end{cases}$$

où $u(t) = -\alpha w_{xt}(1,t)$ désigne le contrôle, $w(x,t)$ le déplacement transversal de la poutre à l'instant t et à la position x. D'un point de vue théorique, le problème est bien posé au sens d'Hadamard, dissipatif et exponentiellement stable. Dans nos travaux, nous établissons tout d'abord, le problème faible associé à ()-() sur des espaces de Hilbert appropriés et nous montrons qu'il admet une unique solution en utilisant des techniques basées sur la méthode de Faedo-Galerkin. Ensuite nous procédons à la semi-discrétisation en espace du système en utilisant la méthode des éléments finis cubiques. Puis on effectue la discrétisation totale en espace-temps de en appliquant la méthode de Crank-Nicholson au schéma semi-discret. Les schémas semi-discret et totalement discret obtenus sont dissipatifs. Enfin, nous proposons des estimations d' erreurs à-priori.

Representation of compact group $SU_q(2)$ in connection with q-Jocobi polynomials

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A unitary representation is a homomorphism of a given group into a group of unitary operators on a Hilbert space. The theory of unitary representation is a natural generalization of classical Fourier analysis. In this talk, we study a finite dimensional unitary representations of q-deformed algebra of continuous functions on the compact group $SU(2)$ namely compact quantum group $SU_q(2)$. The existence of group structure induces the existence noncommutative Hopf algebra on this specific quantum group. We then study the matrix elements of the irreducible unitary representations of this quantum group $SU_q(2)$. We obtain the Peter-Weyl theorem for $SU_q(2)$ and the matrix elements of these unitary representations are explicitly expressed in terms of the little q-Jacobi polynomials which are known as q-analogues of orthogonal polynomials. Using these expressions, the orthogonality relations of these polynomials are obtained in terms of the Haar measure on the quantum group $SU_q(2)$.

Spherical transform of type Delta on the Groupoid

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The study of harmonic analysis on locally compact spaces which are not groups has led some authors to define groupoids as a generalization of groups. In this paper, we present an axiomatic approach which generalizes groups and which has more advantages. We call locally compact groupoids a quadruplet $(\vartheta, s, r, \vartheta^{(0)})$ where ϑ is the total space, $\vartheta^{(0)}$ the basis and s, r the source and goal maps.The idea of studying locally compact groupoids is suggested by: The existence of locally compact spaces which are goupoids and which are not groups. Groupoids preserve all the structures associated with groups. The law is explicit, which allows to define Gelfand pairs and spherical functions. In this article, we define the spherical functions of delta type and give some properties.

Mots clés : Spherical function, Positive type function, Fourier transform, Plancherel formula.

On the L^p-Fourier transform norm for compact extensions of locally compact groups

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Let G be a separable locally compact unimodular group of type I, and $\widehat{G}$ the unitary dual of G endowed with the Mackey Borel structure. We regard the Fourier transform $\mathcal{F}$ as a mapping of $L^1(G)$ to a space of μ-measurable field of bounded operators on $\widehat{G}$ defined for $\pi \in \widehat{G}$ by $L^1(G) \ni f \mapsto \mathcal{F}f : \mathcal{F}f(\pi) = \pi(f)$, where μ denotes the Plancherel measure of G. The mapping $f \mapsto \mathcal{F}f$ extends to a continuous operator $\mathcal{F}^p : L^p(G) \to L^q(\widehat{G})$, where $p \geq 1$ is real number and q its conjugate. We are concerned in this talk with the norm of the linear map $\mathcal{F}^p$. We first record some results on the estimate of this norm for some classes of solvable Lie groups and their compact extensions and discuss the sharpness problem. We look then at the case where G is a separable unimodular locally compact group of type I. Let N be a unimodular closed normal subgroup of G of type I, such that G/N is compact. We show that $\|\mathscr{F}^p(G)\| \leq \|\mathscr{F}^p(N)\|$. In the particular case where $G = K \ltimes N$ is defined by a semi-direct product of a separable unimodular locally compact group N of type I and a compact subgroup K of the automorphism group of N, we show that equality holds if N has a K-invariant sequence $(\varphi_j)_j$ of functions in $L^1(N) \cap L^p(N)$ such that $\|\mathscr{F}\varphi_j\|_q / \|\varphi_j\|_p$ tends to $\|\mathscr{F}^p(N)\|$ when j goes to infinity.

keywords: Lie group, irreducible representation, Fourier transform norm, compact extension, etc

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Using Subordination Principle To Establish The Coefficient Estimates Of A Multivalent Function

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Special functions are some particular mathematical functions which have more or less established names and notations due to their importance in mathematical analysis, functional analysis, physics or other applications. However, there is no

general formal definition, but the list of mathematical functions contains functions which are commonly accepted as special. The theory of special functions has been developed essentially in the nineteenth century. However, in the twentieth century the theory of special functions has been overshadowed by other fields such as real and functional analysis, topology, algebra and differential equations. In this work, the initial coefficient bounds for a class $W_{p,\lambda}(b, S_{\alpha,\beta}(z))$ of an analytic function were obtained. The Fekete-Szegö functional and Hankel determinant for the class were established.

Keywords: Key words and phrases. Analytic functions, Multivalent function, Subordination, Hankel determinant
AMS Mathematics Subject Classification (2010): 30C45, 30C50.

A variational approch to Cauchy problem for nonlinear elliptic systems

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We discuss the Cauchy problem of a class of nonlinear elliptic equations with data on a piece S of the boundary surface by means of a variational approach known in the optimal control literature as “equation error method”. To this end, we use purely nonlinear methods such as successive iterations of a Zaremba-type mixed boundary value problem.

It is about a joint work with B. BELLA, and B. KABORE .

La Conjecture de Heil-Ramanathan-Topiwala (Conjecture HRT)

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En 1996, C. Heil, J. Ramanatha, and P. Topiwala ont postulé que l'ensemble fini $\mathcal{G}(g,\Lambda) = \{e^{2\pi i b_k \cdot} g(\cdot - a_k)\}_{k=1}^N$ est linéairement indépendent pour toute fonction mesurable g dans l'ensemble de Lebesgue $L^2(\mathbb{R})$. Dans cette généralité et à cet jour, la conjecture HRT reste sans solution.

Dans cette communication, nous reviendrons sur l'origine de cette conjecture et nous élaborerons sur les différentes tentatives pour la résoudre. Nous finirons par un aperçu sur une méthode de récurrence introduite afin d'analyser la conjecture. Cette méthode cherche à répondre à la question suivante: Supposons que la conjecture HRT était vraie pour une fonction g et un ensemble de N points $\Lambda = \{(a_k, b_k)\}_{k=1}^N \subset \mathbb{R}^2$. Pour quels autres points $(a,b) \in \mathbb{R}^2 \backslash \Lambda$ pourrions nous conclure que la conjecture demeure vraie pour la même fonction g et le nouvel ensemble de $N+1$ points $\Lambda' = \Lambda \cup \{(a,b)\}$?

A kind of Howe duality between a wreath product and algebra of invariant differential operators

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In this talk, we investigate a kind of Howe duality between generalized symmetric group and algebra of invariant differential operators. In this vein, we study polynomial rings as modules over a ring of invariant differential operators by elaborating its irreducible submodules. The D-module direct image of an irreducible holonomic module with regular singularities under a proper map is semi-simple according to the decomposition theorem. The simplest case is when the map $\pi : X = \text{spec } B \to Y = \text{spec } A$ is finite, and the module is the structure sheaf $B = O_X$. Then an elementary and wholly algebraic proof exists, using essentially the ordinary Galois group G of the function field extension corresponding to π. This proof uses that the irreducible D-submodules of $\pi_+(O_X)$ are in one-to-one correspondence with irreducible representations of G. In this talk, we study the decomposition structure for G equal to the wreath product $G(r,n) = \mathbb{Z}/r\mathbb{Z} \wr \mathcal{S}_n$, where $\mathcal{S}_n$ is the symmetric group of n letters and $\mathbb{Z}/r\mathbb{Z}$ is the cyclic group of order r. . As an application we describe the analytic representation of Olshanetsky-Perelomov operators associated with the weyl group W of type B_n.

Characterizations of the duals of Riesz potentials spaces and homogeneous Sobolev spaces

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We characterize all distributions in the dual $\left(\mathcal{R}^{1,p}(\mathbb{R}^d)\right)^*$ of the Riesz potentials space $\mathcal{R}^{1,p}(\mathbb{R}^d)$ defined as all functions g such that $g = I_1 f$, where f is in the Lebesgue space $L^p(\mathbb{R}^d)$ and I_1 denotes the Riesz potentials operator of order 1. We also characterize all distributions in $\left(\dot{W}^{1,p}(\mathbb{R}^d)\right)^*$, where $\dot{W}^{1,p}(\mathbb{R}^d)$ is the homogeneous Sobolev space defined as the completion of the set $\mathcal{C}_c^\infty(\mathbb{R}^d)$ of all infinitely differentiable and compactly supported functions on $\mathbb{R}^d$, with respect to the Dirichlet norm. Our characterizations are related to the solvability of the equation $div\ F = T$ with distribution T. As a consequence we obtain that the distributions in $\left(\mathcal{R}^{1,p}\right)^*$ coincide with the distributions in $\left(\dot{W}^{1,p}\right)^*$.

A generalization of representation theorems in Hardy-Orlicz spaces on the upper complex half-plane

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We give Poisson and Cauchy representation theorems in Hardy-Orlicz spaces on the upper complex half-plane. We use these theorems for the construction of dual spaces of certain Hardy-Orlicz spaces and also for the characterization of some classical operators in Orlicz spaces.

Large deviations and Berry-Esseen inequalities in the stochastic diffusion driven by a Volterra type process

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We are interested in bounds on the large deviations probability for maximum likelihood estimator of the parameter appearing linearly in the drift of stochastic differential equation driven by a Volterra type process.

Two scale convergence method in Orlicz setting and application

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Fondamental of Two scale convergence and reiterated two scale convergence in Orlicz setting are presented with application to nonlinear degenerated equations with rapidly oscillating coeficient.

On a generalization of the spectral dichotomy method of a matrix with respect to parabolas

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Methods of spectral dichotomy of a matrix which compute spectral projectors on the subspace associated with the eigenvalues external to the parabolas described by a general equation are presented. These methods are modifications of the one proposed in [A. N. Malyshev and M. Sadkane, SIAM J. MATRIX ANAL. APPL. 18 (2), 265-278, 1997] which uses the spectral dichotomy method of a matrix with respect to the imaginary axis. Theoretical and algorithmic aspects of the methods are developed. Numerical results obtained by applying methods presented on matrices are reported. **Keywords:** Spectral dichotomy method, spectral projector, eigensubspaces, eigenvalues.

Time convergence of the Bloch-type model solution

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Keywords: Bloch-type model, density matrix, Pauli's master equation, Strang scheme.

Bloch-type model is a continuous differential system used to modelize time-evolution of energy level occupations in atoms (see [1] and [3]) and in nanostructures such as quantum dots, quantum wells ([5],[6] and [7]). The variable of a Bloch-type model is a matrix called density matrix. Diagonal entries (called populations) are occupation probabilities of the electronic states and off-diagonal entries (called coherences) correspond to transition probabilities from one state to another.
In the litterature, the particular cases of Bloch-type model are studied about two-level quantum model. In our study, we are particulary interested three-level model in taking account the submodels of Pauli's master equation model (relaxation terms) and wave-interaction terms ([4]). In the following, we describe this model as a real nonautonomous dynamical system which preserves the better proprieties such as the trace and the hermicity of density matrix. We show that the solution of sub-equation autonomous converge to a equilibruim solution corresponding to equilibruim of autonomous system and also the full solution is locally bounded in terms of time.
After this, we discretize the Bloch-type model with a version of splitting scheme (Strang's scheme [2]) whose evolution operators are computed by reduction matrices.
In the last part, we are realized the numerical simulations in order to proove the stability and the small CPU time of numerical scheme used. We compare this scheme to a numerical approximation which employed a Taylor serie of matrix exponential.

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Blow-up time of a nonlinear hyperbolic equation with Dirichlet boundary conditions and a potential

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The aim of this paper is the study of the solution of the folowing nonlinear hyperbolic equation subjects to Dirichlet boundary conditions and a potential

$$\phi_{tt}(x,t) = \varepsilon L\phi(x,t) + b(x)g(\phi(x,t)) \quad \text{in} \quad \Omega \times (0,T), \tag{1}$$

$$\phi(x,t) = 0 \quad \text{on} \quad \partial\Omega \times (0,T), \tag{2}$$

$$\phi(x,0) = \phi_0(x) \quad \text{in} \quad \Omega, \tag{3}$$

$$\phi_t(x,0) = 0 \quad \text{in} \quad \Omega, \tag{4}$$

defined in a bounded subset Ω of $\mathbb{R}^N$ with smooth boundary $\partial\Omega$, $\varepsilon >$ is given parameter, $g : [0,\infty) \longrightarrow (0,\infty)$ is a C^1 convex, nondecreasing function, $\int_0^\infty \frac{ds}{g(s)} < \infty$, $\phi_0 \in C^1(\overline{\Omega})$, $\phi_0(x) \geq 0$ in Ω, $b \in C^1(\overline{\Omega})$, $b(x) > 0$ in Ω. We defined the operator L by

$$L\phi = \sum_{i,j=1}^{N} \frac{\partial}{\partial x_i}\left(a_{ij}(x)\frac{\partial \phi}{\partial x_j}\right),$$

where $a_{ij} : \overline{\Omega} \to \mathbb{R}$, $a_{ij} \in C^1(\overline{\Omega})$, $a_{ij} = a_{ji}$, $1 \leq i,j \leq N$, and there exists a constant $C > 0$ such that

$$\sum_{i,j=1}^{N} a_{ij}(x)\xi_i\xi_j \geq C\|\xi\|^2 \; \forall x \in \overline{\Omega} \; \forall \xi = (\xi_1, ..., \xi_N) \in \mathbb{R}^N,$$

where $\|\cdot\|$ is the Euclidean norm of $\mathbb{R}^N$. Here $(0, T)$ is the maximal time interval of existence of the solution ϕ.
Under suitable hypotheses, we show that the solution of the above problem blows up in a finite time, and its blow-up time tends to the one of the solution of a differential equation when a certain parameter goes to zero. We give some numerical results to illustrate our analysis at the end of our study.

`AMS subject classification(2020):` *35B40, 35B50, 35L20, 35L70, 65M06.*

`Key words and phrases:` *Nonlinear hyperbolic equation, blow-up, convergence, numerical blow-up time.*

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In this study we perform a modal analysis of the linear inviscid shallow water equations using a non constant bathymetry, continuous and discontinuous Galerkin approximations. By extracting the discrete eigenvalues of the resulting algebraic linear system written on the form of a generalized eigenvalue / eigenvector problem we first show that the regular variation of the bathymetry does not prevent the presence of spurious inertial modes when centered finite element pairs are used. Secondly, we show that such spurious modes are not present in discontinuous Galerkin discretizations when all variables are approximated in the same discrete space. Such spurious inertial modes have been found very damageable for the quality of inertia-gravity and Rossby modes in ocean modelling.

Keywords : Discontinous Galerkin - mixed finite element - Shallow water equations - Spurious modes - variable bathymetry.

Convolution in Orlicz spaces on hypergroup

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Hypergroups generalize locally compact groups. They appear when the Banach space of all bounded Radon measures on a locally compact space carries a convolution having all properties of a group convolution apart from the fact that the convolution of two point measures is a probability measure with compact support and not necessarily a point measure. The intention was to unify harmonic analysis on duals of compact groups, double coset spaces $G//H$ (H a compact subgroup of a locally compact group G), and commutative convolution algebras associated with product linearization formulas of special functions. The notion of hypergroup has been sufficiently studied . Harmonic analysis and probability theory on commutative hypergroups are well developed meanwhile where many results from group theory remain valid. When G is a commutative hypergroup, the convolution algebra $M_c(G)$ consisting of measures with compact support on G is commutative. The typical example of commutative hypergroup is the double coset $G//K$ when G is a locally compact group, K is a compact subgroup of G such that (G, K) is a Gelfand pair.

The well-known L^p-conjecture stated that if $1 < p < \infty$ and G is a locally compact group, then the Lebesgue space $L^p(G)$ is a Banach algebra under the convolution product if and only if G is compact. In their paper,Abtahi; Nasr-Isfahani and Rejali mentioned that for each $p > 2$, if $f * g$ exists a.e. for all f, g $\in L^p(G)$, then G is compact, and so automatically $f * g \in L^p(G)$. In the setting of hypergroups, this result was studied by Tabatabaie and Haghighifarin under some conditions. Assuming that G is a commutative hypergroups, Salec; Kumar and Tabatabaie give a necessary condition in terms of aperiodic elements of the center of G, for the convolution $f * g$ to exist a.e., where f and g are arbitrary elements of Orlicz spaces $L^{\Phi_1}(G)$and $L^{\Phi_2}(G)$, respectively. When the hypergroup G is not commutative, it is possible to involve a compact sub-hypergroup K of G leading to a commutative subalgebra of $M_c(G)$. In fact, if K is a compact sub-hypergroup of a hypergroup G, the pair (G, K) is said to be a Gelfand pair if $M_c(G//K)$ the convolution algebra of measures with compact support on $G//K$ is commutative. The notion of Gelfand pairs for hypergroups is well-known.

The goal of this paper is to study the condition for the existence of the convolution of two functions each coming from an Orlicz space on a hypergroup which is not necessary commutative, but such that (G, K) is a Gelfand pair. For this, we have to characterize the center $Z(G)$ of the hypergroup G, then describe its action on G, and finally we establish our main result, namely the existence of the convolution of two Orlics spaces in terms of aperiodic elements of $Z(G)$:

Theorem

Let (G, K) be a Gelfand pair and (Φ_1, Φ_2) be a pair of Young functions which satisfies the sequence condition. If for each $f \in L^{\Phi_1}(G)$ and $g \in L^{\Phi_2}(G)$, $(f * g)(x)$ exists for almost every $x \in G$, then the set of aperiodic elements of $Z(G)$ with respect to the action $Z(G) \curvearrowright (G, \mu_G)$ is empty.

Late-time asymptotic behavior of hyperbolic systems with stiff relaxation source term

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We investigate the late-time asymptotic behavior of solutions to nonlinear hyperbolic systems of conservation laws containing stiff relaxation terms. To adress such an issue, we introduce an asymptotic expansion and derive an effective system of equations describing the late-time/stiff relaxation singular limit. The structure of this new system is discussed and the role of a mathematical entropy is emphasized.